Compactification, Vacuum Energy and Quintessence[1]

M. C. Bento and O. Bertolami

Instituto Superior Técnico, Departamento de Física,

Av. Rovisco Pais 1, 1096 Lisboa Codex, Portugal

Abstract

We study the possibility that the vacuum energy density of scalar and internal-space gauge fields arising from the process of dimensional reduction of higher dimensional gravity theories plays the role of quintessence. We show that, for the multidimensional Einstein-Yang-Mills system compactified on a $\mathbf{R} \times S^3 \times S^d$ topology, there are classically stable solutions such that the observed accelerated expansion of the Universe at present can be accounted for without upsetting structure formation scenarios or violating observational bounds on the vacuum energy density.

---

[1] Third Award in the 1999 Essay Competition of the Gravity Research Foundation

1   Introduction

Recently, strong evidence is emerging that the Universe is dominated by a smooth component with an effective negative pressure and expanding in an accelerated fashion. These findings arise from the study of more than 50 recently discovered Type IA Supernovae with red-shifts greater than $z \geq 0.35$ [1]. Such studies, carried out by two different groups [1, 2], lead to the striking result that the deceleration parameter

$$q_0 \equiv -\frac{\ddot{a}\,a}{\dot{a}^2} \; , \tag{1}$$

where $a(t)$ is the scale factor, is negative

$$-1 \lesssim q_0 \; < \; 0 \; . \tag{2}$$

It follows from the Friedmann and Raychaudhuri equations for an homogeneous and isotropic geometry that, if the sources driving the expansion are vacuum energy and matter, with equation of state $p = \omega\rho, \; -1 \leq \omega \leq 0$, then the deceleration parameter is given by:

$$q_0 = \frac{1}{2}(3\omega + 1)\Omega_M - \Omega_\Lambda \; , \tag{3}$$

where $\Omega_{M(V)}$ denotes the energy density of matter (vacuum) in units of the critical density. For a Universe where the matter component is dominated by non-relativistic matter or dust, $\omega = 0$, and therefore the combination $\Omega_M \sim 0.4$ and $\Omega_\Lambda \sim 0.7$ seems observationally favoured. Of course, the value $\Omega_\Lambda \sim 0.7$, although consistent with observation (see Ref. [3] for a list of the important constraints), implies a quite unnatural fine tuning of parameters if it arises from any known particle physics setting (see Refs. [4] for a thorough review and Refs. [3, 5] for possible connections with fundamental symmetries like Lorentz invariance and S-duality in string theories). Furthermore, Ω_M and Ω_Λ of the same order suggests that we live in a rather special cosmological period.

While the most straightforward candidate for a smooth component is a cosmological constant, a plausible alternative is a dynamical vacuum energy, or "quintessence".

Suggestions along these lines have been proposed a long time ago [6], although yielding a vanishing deceleration parameter. A number of quintessence models have been put forward, the most popular of which invoke a scalar field with a very shallow potential, which until recently was overdamped in its evolution by the expansion of the Universe, allowing for its energy density to be smaller than the radiation energy density at early times, such that at present $\Omega_M \lesssim \Omega_\Lambda$ [7, 8]. It was also shown that scalar fields with an exponential type potential can, under conditions, render a negative q_0 [9, 10]. Other suggestions include the string theory dilaton together with gaugino condensation [11], an axion with an almost massless quark [12], a time-dependent vacuum energy induced by D-particle recoil [13], etc. However interesting, most of these suggestions necessarily involve a quite severe fine tuning of parameters [14]. This fact calls for constructions that allow for a negative deceleration parameter using sources of quintessence that ideally do not require a potential. In this context, it has been shown that a scalar field coupled with gravity non-minimally, namely a self-interacting Brans-Dicke type field with a negative coupling, can be used for this purpose [15].

In this work, we study the possibility that scalar fields arising from the process of dimensional reduction of higher dimensional gravity theories, together with internal gauge fields, play the role of quintessence. The stability of the required compactification of the extra dimensions is related with the dynamics of these fields. Classical and quantum stabilility depends on the existence of minima of the relevant potential that are classically or, at least, semiclassically stable. We show that, for the multidimensional Einstein-Yang-Mills system [16, 17, 18], the cosmological framework following from demanding that compactifying solutions are classically stable can also be used to drive an accelerated expansion at present, at the expense of the contribution of higher dimensional fields. In fact, the stability of compactification requires fine tuning the higher-dimensional cosmological constant meaning that, in this respect, our proposal is afflicted with the same difficulty of other quintessence models. Nevertheless, the most advantageous aspects of our setting are that it rests on the fruitful ground of the Einstein-Yang-Mills system and, therefore, there is no need to postulate *ad hoc* potentials and also that the dimensional reduction procedure determines, via the

theory of symmetric fields, the cosmological model unambiguously. Thus, in a single framework, the issues of compactification and accelerated expansion of the Universe are related and the ground-state energy of fields emerging from the compactification scenario can actually be regarded as a consistent quintessence candidate, a scenario that we choose to call "quintessential compactification".

2   The Generalized Kaluza-Klein Model

Compactification is a crucial step in rendering multidimensional theories of unification, such as generalized Kaluza–Klein theories, Supergravity and Superstring theories, consistent with our four-dimensional world. Phenomenology requires that the extra dimensions are stable and Planck size (see, however, [19] for a different proposal concerning this issue). A necessary condition for the stability of the extra dimensions is the presence of matter with repulsive stresses to counterbalance gravity. Magnetic monopoles [20], Casimir forces [21] and Yang-Mills fields [16, 17] have been suggested for this purpose. The case of Yang-Mills fields is particularly interesting as it illustrates the importance of considering non-vanishing internal as well as external-space components of the gauge fields. Indeed, as shown in Ref. [17], it is this feature that renders compactifying solutions classically as well as semiclassically stable. Moreover, it was shown in Ref. [18], using the quantum cosmology formalism, that, for the Einstein-Yang-Mills system, compactifying solutions with non-vanishing external-space components of the gauge field are correlated with the expansion of the Universe. This implies that, for expanding universes, it is more likely that stable compactification solutions arise.

Following [16, 17], we consider an $SO(N)$ gauge field with $N \geq 3 + d$ in $D = 4 + d$ dimensions and an homogeneous and (partially) isotropic spacetime in a $\mathbf{R} \times S^3 \times S^d$ topology. The relevant coset compactification splitting of the D-dimensional spacetime M^D is the following

$$M^D = \mathbf{R} \times G^{\text{ext}}/H^{\text{ext}} \times G^{\text{int}}/H^{\text{int}} \ , \tag{4}$$

where $\mathbf{R}$ denotes the timelike direction, $G^{\text{ext(int)}} = SO(4)(SO(d+1))$ and $H^{\text{ext(int)}} = SO(3)(SO(d))$ are respectively the homogeneity and isotropy groups in $3(d)$ dimensions. For the multidimensional Einstein-Yang-Mills model we consider, the gauge field has only time-dependent spatial components on the 3-dimensional physical space.

The model is derived from the generalized Kaluza-Klein action:

$$S[\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{A}_{\hat{\mu}}, \hat{\chi}] = S_{\text{gr}}[\hat{g}_{\hat{\mu}\hat{\nu}}] + S_{\text{gf}}[\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{A}_{\hat{\mu}}] + S_{\text{inf}}[\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{\chi}] \tag{5}$$

with

$$S_{\text{gr}}[\hat{g}_{\hat{\mu}\hat{\nu}}] = \frac{1}{16\pi\hat{k}} \int_{M^D} d\hat{x}\sqrt{-\hat{g}} \left(\hat{R} - 2\hat{\Lambda}\right) , \tag{6}$$

$$S_{\text{gf}}[\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{A}_{\hat{\mu}}] = \frac{1}{8\hat{e}^2} \int_{M^D} d\hat{x}\sqrt{-\hat{g}} \, \text{Tr}\hat{F}_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}} , \tag{7}$$

$$S_{\text{inf}}[\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{\chi}] = -\int_{M^D} d\hat{x}\sqrt{-\hat{g}} \left[\frac{1}{2}(\partial_{\hat{\mu}}\hat{\chi})^2 + \hat{U}(\hat{\chi})\right] , \tag{8}$$

where $\hat{g}$ is $\det(\hat{g}_{\hat{\mu}\hat{\nu}})$, $\hat{g}_{\hat{\mu}\hat{\nu}}$ is the D-dimensional metric, $\hat{R}$, $\hat{e}$, $\hat{k}$ and $\hat{\Lambda}$ are, respectively, the scalar curvature, gauge coupling, gravitational and cosmological constants in D dimensions. In addition, the following field variables are defined in M^D: $\hat{F}_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}}\hat{A}_{\hat{\nu}} - \partial_{\hat{\nu}}\hat{A}_{\hat{\mu}} + \left[\hat{A}_{\hat{\mu}}, \hat{A}_{\hat{\nu}}\right]$ is the gauge field strength, $\hat{A}_{\mu}$ denotes the components of the gauge field and $\hat{\chi}$ is the inflaton, responsible for the inflationary expansion of the external space. The inflaton potential, $\hat{U}(\hat{\chi})$, is taken to be bounded from below, having a global minimum so that $\hat{U}_{\min} = 0$.

We consider vacuum solutions where the splitting of internal and external dimensions of spacetime corresponds to a factorization in a product of spaces

$$M^D = M^4 \times K^d , \tag{9}$$

M^4 being the four-dimensional Minkowski spacetime, K^d a Planck-size $d-$dimensional compact space. We assume that $M^D = \mathbf{R} \times S^3 \times S^d$, where S^3 and S^d are 3 and d-dimensional spheres.

The spatially homogeneous and (partially) isotropic field configurations relevant for our cosmological model are symmetric under the action of the group $G^{\text{ext}} \times G^{\text{int}}$. The following realization of M^D can then be constructed [17]

$$
\begin{aligned}
M^D &= \mathbf{R} \times SO(4)/SO(3) \times SO(d+1)/SO(d) \\
&= \mathbf{R} \times [SO(4) \times SO(d+1)]/[SO(3) \times SO(d)] \ .
\end{aligned}
\tag{10}
$$

The metric corresponding to the D-dimensional spacetime is given by

$$
ds^2 = -\tilde{N}^2(t)dt^2 + \tilde{a}^2 d\Omega_3^2 + b^2(t)d\Omega_d^2 \ ,
\tag{11}
$$

where $\tilde{a}(t)$ and $b(t)$ are the scale factors of S^3 and S^d respectively, and $\tilde{N}(t)$ is the lapse function.

The remaining field configurations associated with the above geometry, described in Ref. [17], are built using the theory of symmetric fields (see eg. [22] and references therein). Substituting the corresponding Ansätze into the action (5) and performing the conformal transformations

$$
\tilde{N}^2(t) = \left[\frac{\langle b \rangle}{b(t)}\right]^d N^2(t) \ ,
\tag{12}
$$

$$
\tilde{a}^2(t) = \left[\frac{\langle b \rangle}{b(t)}\right]^d a^2(t) \ ,
\tag{13}
$$

where $\langle b \rangle$ is the vacuum expectation value of $b(t)$, we obtain a one-dimensional effective reduced action [17]:

$$
\begin{aligned}
S_{\text{eff}} &= 16\pi^2 \int dt N a^3 \left\{ -\frac{3}{8\pi k}\frac{1}{a^2}\left[\frac{\dot{a}}{N}\right]^2 + \frac{3}{32\pi k}\frac{1}{a^2} + \frac{1}{2}\left[\frac{\dot{\psi}}{N}\right]^2 + \frac{1}{2}\left[\frac{\dot{\chi}}{N}\right]^2 \right. \\
&\quad + \left. e^{d\beta\psi}\frac{3}{4e^2}\frac{1}{a^2}\left(\frac{1}{2}\left[\frac{\dot{f}_0}{N}\right]^2 + \frac{1}{2}\left[\frac{\mathcal{D}_t\mathbf{f}}{N}\right]^2\right) + e^{-2\beta\psi}\frac{d}{4e^2}\frac{1}{\langle b \rangle^2}\frac{1}{2}\left[\frac{\mathcal{D}_t\mathbf{g}}{N}\right]^2 - W \right\} ,
\end{aligned}
\tag{14}
$$

where $k = \hat{k}/v_d \langle b \rangle^d$ is Newton's constant, $e^2 = \hat{e}^2/v_d\langle b \rangle^d$, e being the gauge coupling, $\beta = \sqrt{16\pi k/d(d+2)}$ and v_d is the the volume of S^d for $b = 1$. Moreover, we have set $\psi \equiv \beta^{-1}\ln(b/\langle b \rangle)$ and $\chi \equiv \sqrt{v_d\langle b \rangle^d}\hat{\chi}$. The dots denote time derivatives and $\mathcal{D}_t$ is the covariant derivative with respect to the remaining $SO(N-3-d)$ gauge field $\hat{B}(t)$ in $\mathbf{R}$:

$$
\mathcal{D}_t\mathbf{f}(t) = \frac{d}{dt}\mathbf{f(t)} + \hat{B}(t)\mathbf{f(t)} \ , \ \mathcal{D}_t\mathbf{g}(t) = \frac{d}{dt}\mathbf{g(t)} + \hat{B}(t)\mathbf{g(t)} \ .
\tag{15}
$$

It is important to point out that $f_0(t), \mathbf{f} = \{f_p\}$ represent the gauge field components in 4-dimensional physical space-time, while $\mathbf{g} = \{g_q\}$ denotes the components in K^d, $\hat{B}$ is an $(N-3-d) \times (N-3-d)$ antisymmetric matrix and ψ is the scalar field emerging from the compactification procedure.

The potential W, in (14), is given by

$$
\begin{aligned}
W &= e^{-d\beta\psi} \left[-e^{-2\beta\psi} \frac{1}{16\pi k} \frac{d(d-1)}{4} \frac{1}{\langle b \rangle^2} + e^{-4\beta\psi} \frac{1}{\langle b \rangle^4} \frac{d(d-1)}{8e^2} V_2(\mathbf{g}) + \frac{\Lambda}{8\pi k} + U(\chi) \right] \\
&+ e^{-2\beta\psi} \frac{1}{(a\langle b \rangle)^2} \frac{3d}{32e^2} (\mathbf{f} \cdot \mathbf{g})^2 + e^{d\beta\psi} \frac{3}{4e^2 a^4} V_1(f_0, \mathbf{f}) \ ,
\end{aligned}
\tag{16}
$$

where $\Lambda = v_d \langle b \rangle^d \hat{\Lambda}$, $U(\chi) = v_d \langle b \rangle^d \hat{U} \left(\hat{\chi}/\sqrt{v_d b_0^d} \right)$ and

$$
\begin{aligned}
V_1(f_0, \mathbf{f}) &= \frac{1}{8} \left[\left(f_0^2 + \mathbf{f}^2 - 1 \right)^2 + 4 f_0^2 \mathbf{f}^2 \right] \ , \tag{17} \\
V_2(\mathbf{g}) &= \frac{1}{8} \left(\mathbf{g}^2 - 1 \right)^2 \ , \tag{18}
\end{aligned}
$$

are related with the external and internal components of the gauge fields, respectively. Variables N and $\hat{B}$ are Lagrange multipliers associated with the symmetries of the effective action (14). The lapse function N is related to the invariance of S_{eff} under arbitrary time reparametrizations, while $\hat{B}$ is connected with the local remnant $SO(N-d-3)$ gauge invariance. Without loss of generality the gauge $N = 1$ will be used in what follows. The equations of motion for the physical variables $a, \psi, \chi, f_0, \mathbf{f}, \mathbf{g}$ can be found in Ref. [17].

The Friedmann equation and the equation of motion for the field ψ, relevant for our quintessence proposal are the following:

$$
\left(\frac{\dot{a}}{a} \right)^2 = -\frac{1}{4a^2} + \frac{8\pi k}{3} \left[\frac{\dot{\psi}^2}{2} + W(a, \psi) + \rho \right] \ , \tag{19}
$$

$$
\ddot{\psi} + 3 \left(\frac{\dot{a}}{a} \right) \dot{\psi} + \frac{\partial W}{\partial \psi} = 0 \ . \tag{20}
$$

Notice that we have added a term corresponding to the matter energy density since its contribution is quite important for the late time Universe, $\rho = \rho_0 (\frac{a_0}{a})^3$, where ρ_0 and a_0 are the matter energy density and the scale factor at present, respectively.

The compactification scenario we envisage involves static vacuum configurations of the gauge and inflaton fields:

$$f_0 = f_0^v, \ \mathbf{f} = \mathbf{f}^v, \ \mathbf{g} = \mathbf{g}^v = \mathbf{0}, \ \chi = \chi^v \ . \tag{21}$$

We also assume that $\mathbf{f}$ and $\mathbf{g}$ are orthogonal and that $U(\chi^v) = 0$. For simplicity, we use the notation $v_1 \equiv V_1(f_0^v, \mathbf{f}^v)$ and $v_2 \equiv V_2(\mathbf{g}^v) = \frac{1}{8}$ in what follows. The potential (16) simplifies then to

$$W = e^{-d\beta\psi} \left[-e^{-2\beta\psi} \frac{1}{16\pi k} \frac{d(d-1)}{4} \frac{1}{\langle b \rangle^2} + e^{-4\beta\psi} \frac{1}{\langle b \rangle^4} \frac{d(d-1)}{8e^2} v_2 + \frac{\Lambda}{8\pi k} \right] + e^{d\beta\psi} \frac{3}{4e^2 a^4} v_1 \ , \tag{22}$$

where the last term arises from the contribution of the external-space components of the gauge field and clearly represents the contribution of radiation for the energy density of the Universe.

As discussed in Ref. [17], different values for the cosmological constant Λ correspond to different compactification scenarios. Indeed, for $\Lambda > c_2/16\pi k$, where $c_2 = [(d+2)^2(d-1)/(d+4)]e^2/16v_2$, there are no compactifying solutions and for

$$\frac{c_1}{16\pi k} < \Lambda < \frac{c_2}{16\pi k} \ , \tag{23}$$

with $c_1 = d(d-1)e^2/16v_2$, a compactifying solution exists which is classically stable but semiclassically unstable. On the other hand, a value of $\Lambda < c_1/16\pi k$ implies that the effective 4-dimensional cosmological constant, $\Lambda^{(4)} = 8\pi k W(a \to \infty, \psi)$, is negative. As $\Lambda^{(4)}$ must satisfy the order of magnitude observational bound

$$\Lambda^{(4)} \approx 10^{-120} \frac{1}{16\pi k} \ , \tag{24}$$

we consider the following fine-tuning of the multidimensional cosmological constant $\Lambda = c_1(1+\delta)/16\pi k$, where δ is clearly proportional to $\Lambda^{(4)}$. On the other hand, since we are interested in compactifying solutions, for which $\psi \approx 0$, we choose Λ such that $\psi = 0$ corresponds to the absolute minimum of (22), where $\langle b \rangle^2 = 16\pi k v_2/e^2$. Hence

$$\Lambda = \frac{d(d-1)}{16\langle b \rangle^2} (1+\delta) \ . \tag{25}$$

8

Substituting (25) in (22), we obtain, for large a (implying that the radiation term can be neglected)

$$W = \frac{d(d-1)}{128\pi k\langle b\rangle^2}\delta \quad .$$
(26)

Of course, a non-vanishing δ introduces a semiclassical instability in our compactification solution; however, the decay rate of the compactified vacuum is such that the decompactification time is much greater, by many orders of magnitude, than the age of the Universe.

Naturally, compactification occurs prior inflation, which is driven by the potential $U(\chi)$ of the inflaton, χ. It should be pointed out that the inflaton itself could, via a suitable choice of its potential or by relaxing the condition $U(\chi^v) = 0$, be at the origin of a late accelerated expansion. This possibility has been proposed some time ago and was called "deflation" [23] or "quintessential inflation" in a more recent version of the idea [24]. This is certainly an interesting suggestion that can actually be implemented in the context of our model. Interestingly, quintessential inflation can be, in principle, detected. Indeed, as discussed in Ref. [24], a distinct feature of quintessential inflation is the form of its gravitational wave spectrum, which although inacessible to gravity wave detectors under construction, such as LIGO and VIRGO, may be within reach for a future generation of detectors.

3   The Deceleration Parameter in Quintessential Compactification

Let us now compute the deceleration parameter for late times in the context of our quitessential compactification scenario. We differentiate the Friedmann equation (19) and substitute the resulting term in $\ddot{\psi}$ by eq. (20), to obtain

$$\ddot{a} = \frac{8\pi k}{3}a\left[\frac{\dot{\psi}^2}{2} + W(a,\psi)\right] - \frac{4\pi k}{3a^2}\rho_0 a_0^3 + \frac{4\pi k}{3\dot{a}}a^2\left[-3H\dot{\psi}^2 + \frac{\partial W}{\partial a}\dot{a}\right] \quad ,$$
(27)

where $H = \frac{\dot{a}}{a}$. Neglecting the last term, a very good approximation since $\frac{\partial W}{\partial a} \sim a^{-5}$, we have

$$\ddot{a} = -\frac{8\pi k}{3} a \left[\dot{\psi}^2 - W(a, \psi) + \frac{\rho_0 a_0^3}{2a^3} \right] \quad . \tag{28}$$

For $\psi = 0$ [2], we get for the deceleration parameter, substituting (26) in (28) and setting $\dot{\psi} = 0$

$$q = \frac{-\frac{a^2}{\langle b \rangle^2} \hat{\delta} + \frac{4\pi k}{3} \frac{\rho_0 a_0^3}{a}}{-\frac{1}{4} + \frac{a^2}{\langle b \rangle^2} \hat{\delta} + \frac{8\pi k}{3} \frac{\rho_0 a_0^3}{a}} \quad , \tag{29}$$

where $\hat{\delta} = \frac{d(d-1)}{48} \delta$. Given the bound (24), then $\delta = \delta_0 10^{-120}$, where δ_0 will be computed below. On the other hand, taking the gauge coupling $e \sim 0.3$, the radius of the compact manifold K^d is about 10 times greater than the Planck size [3], and $\left(\frac{a_0}{\langle b \rangle} \right)^2 = \alpha_0 10^{120}$, where α_0 is an order one constant. Hence, we obtain for the deceleration parameter at present

$$q_0 = \frac{-\delta_1 + \frac{\epsilon}{2}}{-\frac{1}{4} + \delta_1 + \epsilon} \quad , \tag{30}$$

where $\delta_1 \equiv d(d-1)\alpha_0\delta_0/48$ and

$$\epsilon \equiv \frac{8\pi k}{3} \rho_0 a_0^2 = \frac{320\pi}{3} \alpha_0 \Omega_M h_0^2 \quad , \tag{31}$$

where h_0 parametrizes the observational uncertainty in the Hubble constant, $H_0 = 100 \, h_0 \, km \, s^{-1} \, Mpc^{-1}$ and $0.4 \lesssim h_0 \lesssim 0.7$.

In order to obtain a bound on δ_0, we compute the time t_q when quintessence states started dominating the dynamics of the Universe. The value of $a_q = a(t_q)$ can be estimated equating the contributions of W and $\rho(a_q)$. From the observational bound $\Omega_M \lesssim 0.3$ [25], we obtain:

---

[2] Assuming that ψ oscillates around the minimum, i.e. $W \sim \psi^2$, we then have, according to the virial theorem, $< \dot{\psi}^2 > = < W >$. In this case, we would obtain a positive q_0. However, since the minimum is very deep and the time scale of the problem is the Planck time, it is reasonable to assume that ψ has had enough time to settle at the minimum of the potential.

[3] It is interesting to notice that, if $\langle b \rangle$ were much greater than the Planck size, as suggested in [19], it would render our quitessence proposal untenable as the order of magnitude of δ is determined by (24).

$$\alpha^3 \, \delta_0 \lesssim \frac{1536 \, \pi}{d(d-1)} \, h_0^2 \ , \tag{32}$$

where $\alpha \equiv \frac{a_q}{a_0}$. Since the red-shift of the supernova data used to infer the accelerated expansion of the Universe is $z \geq 0.35$, then $\alpha \leq 0.74$ and, for $d = 7$ and $h_0 = 0.5$, we get

$$\delta_1 \lesssim 71 \ , \tag{33}$$

which implies, for e.g. $\delta_1 = 70$, that

$$q_0 = -0.56 \ , \tag{34}$$

which sits inside the most likely region of values for q_0, as revealed by observational data [1, 2].

Notice that, with $\alpha \leq 0.74$, the quintessence domination period is rather recent in the history of the Universe and, hence, structure formation scenarios, for which the condensation period is $z_c \gtrsim 10$, given the most recent Hubble deep field surveys, are unaffected by our quintessence proposal provided the vacuum energy density is not too large. It is easy to show that this is indeed the case as, even for $z \approx 0.35$, the time when quintessence starts dominating the dynamics, $\Omega_V(z \geq 0.35) \simeq 0.5$, consistent with bounds arising from anthropic considerations which imply that, for a flat Universe (which is not the case of our model), a vacuum energy no greater than about $\pi(1 + z_c)^3 \rho_0$ does not prevent gravitational condensation [4]. At present, one should expect $\rho_V \lesssim 3 \, \rho_0$ [26]. Our model is compatible with this bound and also with the upper limit arising from gravitational lensing studies, namely that, at present, $\Omega_V < 0.75$ [27, 28].

A distinct feature of the model is that, in spite of having a closed topology, a phase of accelerated expansion can take place. Indeed, writing the Friedmann equation as

$$\dot{a}^2 + V(a) = -\frac{1}{4} \ , \tag{35}$$

with

$$V(a) = -\frac{a^2}{\langle b\rangle^2}\hat{\delta} - \frac{8\pi k}{3}\frac{\rho_0 a_0^3}{a} \quad , \tag{36}$$

we see that accelerated expansion can be achieved provided

$$V(a_M) \; < \; -\frac{1}{4} \quad , \tag{37}$$

where a_M is the maximum of $V(a)$. This requirement implies the condition

$$4\,\delta_1\,\mu^2 + \frac{\epsilon}{\mu} - 1 > 0 \quad , \tag{38}$$

where $\mu \equiv \frac{a_M}{a_0}$. It can easily be verified that, for $\delta_1 = 70$ and $\alpha_0 = \mathcal{O}(1)$, this inequality holds for any μ, hence representing a consistency check for our proposal. Clearly, our model presents, however brief, a coasting period where $a \sim constant$.

4 Discussion and Conclusions

In this work, we have proposed a cosmological model based on the multidimensional Einstein-Yang-Mills system, compactified on $\mathbf{R} \times S^3 \times S^d$ spacetime. We have shown that the very fine tuning on the higher dimensional cosmological constant needed for stable compactifying solutions and rendering the vacuum energy density consistent with observational bounds, can also account for the observed accelerated expansion of the Universe, for reasonable values of the model parameters. Specifically, we obtain a deceleration parameter $q_0 = -0.56$. This is achieved via the vacuum contribution of a scalar field, very much like in the so-called quintessence scenarios but, in our model, an internal gauge field is also involved and these fields arise from the compactification process via the dimensional reduction procedure. Furthermore, we have shown that, since the quintessence domination period is quite recent in the history of the Universe, known scenarios for structure formation remain unaffected by our quintessence proposal and bounds on the vacuum energy density are respected.

Finally, it is interesting to point out that our setting allows, quite naturally, for a quintessential inflationary extension, and although we have chosen here to study the "minimal" version of the cosmological model arising from the multidimensional

Einstein-Yang-Mills theory, further work on "quintessential compactification - inflation" follows immediately from the model.

References

[1] S.J. Perlmutter et al. (The Supernova Cosmology Project), "Measurement of the Cosmological Parameter Ω and Λ from the First 7 Supernovae at $z \geq 0.35$", astro-ph/9608192; *Nature* **391** (1998) 51.

[2] A.G. Riess et al., "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant", astro-ph/9805201;

P.M. Garnavich et al., "Supernova Limits on the Cosmic Equation of State", astro-ph/9806396.

[3] O. Bertolami, *Class. Quantum Gravity* **14** (1997) 2748.

[4] S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1;

"Theories of the Cosmological Constant", astro-ph/9610044.

[5] O. Bertolami and R. Schiappa, "Modular Quantum Cosmology", gr-qc/9810013; M.C. Bento and O. Bertolami, *Gen. Relativity and Gravitation* **28** (1996) 565.

[6] O. Bertolami, *Il Nuovo Cimento* **93B** (1986) 36; *Fortschr. Physik* **34** (1986) 829.

[7] I. Zlatev and L. Wang and P. Steinhardt, "Quintessence, Cosmic Coincidence, and the Cosmological Constant", astro-ph/9807002.

[8] R.R. Caldwell, R. Dave and P.J. Steinhardt, *Phys. Rev. Lett.* **80** (1998) 1582.

[9] P.J.E. Peebles and B. Ratra, *Ap. J. Lett.* **325** (1988) 117.

[10] P.G. Ferreira and M. Joyce, *Phys. Rev.* **D58** (1998) 023503.

[11] P. Binétruy, "Models of Dynamical Supersymmetry Breaking and Quintessence", hep-ph/9810553.

[12] J.E. Kim, "Axion and Almost Massless Quark as Ingredients of Quintessence", hep-ph/9811509.

[13] J.Ellis, N.E. Mavromatos and D.V. Nanopoulos, "Time-Dependent Vacuum Energy Induced by D-Particle Recoil", hep-ph/9810086.

[14] C. Kolda and D. Lyth, "Quintessential Difficulties", hep-ph/9811375.

[15] O. Bertolami and P.J. Martins, "Non-minimal coupling and Quintessence", Lisbon Preprint DF/IST-2.99.

[16] Yu.A. Kubyshin, V.A. Rubakov and I.I. Tkachev, *Int. J. Mod. Phys.* **A4** (1989) 1409.

[17] O. Bertolami, Yu.A. Kubyshin and J.M. Mourão, *Phys. Rev.* **D45** (1992) 3405.

[18] O. Bertolami, P.D. Fonseca and P.V. Moniz, *Phys. Rev.* **D56** (1997) 4530.

[19] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, "Phenomenology, Astrophysics and Cosmology of Theories with Sub-Millimeter Dimensions and TeV Scale Quantum Gravity", hep-ph/9807344.

[20] S. Randjbar-Daemi, A. Salam and J. Strathdee, *Nucl. Phys.* **B214** (1983) 491.

[21] T. Appelquist and A. Chodos, *Phys. Rev. Lett.* **50** (1983) 141;
P. Candelas and S. Weinberg, *Nucl. Phys.* **B237** (1984) 397.

[22] O. Bertolami, J.M. Mourão, R.F. Picken and I.P. Volobujev, *Int. J. Mod. Phys.* **A6** (1991) 4149.

[23] B. Spokoiny, *Phys. Lett.* **B315** (1993) 40.

[24] P.J.E. Peebles and A. Vilenkin, "Quintessential Inflation", astro-ph/9810509.

[25] N. Bahcall, "Cosmology with Clusters of Galaxies", astro-ph/9812076.

[26] H. Martel, P.R. Shapiro and S. Weinberg, "Likely Values of the Cosmological Constant", astro-ph/9701099.

[27] M. Fukugita and E.L. Turner, *Mon. Not. R. Ast. Soc.* **253** (1991) 99.

[28] D. Maoz and H-W. Rix, *Ap. J.* **416** (1993) 425.